B48 2002-3

DAVID PEARCE

LECTURES NOVEMBER 12, 14: PART 2 OF 2

THE THEORY OF EXHAUSTIBLE RESOURCE DEPLETION

Theory and practice

The previous lecture suggested that price might be a reasonable indicator of resource scarcity, and that, since minerals and energy prices tended to go down over time, there appear to be no ‘limits to growth’, at least not in the near-term. The caveats surrounding this conclusion were noted.

But what would theory predict with reference to the path of the price of natural resources? The theory would generally predict rising prices as the resource gets scarcer, i.e. as we approach T - we show this shortly. These rising prices would signal the scarcity of the resource and this signalling function has two purposes. First, it would act as an indicator of scarcity – if price is falling perhaps we have no need to worry. If it is rising, we should start worrying. Second, the higher price sends a signal that the resource whose price is rising will be substituted for by some other resource, perhaps another exhaustible resource, perhaps a renewable resource. Rising prices will also make recycling ‘old’ materials more worthwhile than before because the price of ‘primary’ materials will rise relative to the price of ‘secondary’ (scrap) materials. Rising prices will also stimulate more effort at discovering new resources, and they will encourage people to be more efficient in their use of materials, i.e. the ratio of resource use to ‘output’ will decline. (Note that energy cannot be recycled so the recycling effect is not applicable to energy).

Overall, then, so long as markets work in such a way that prices reflect scarcity, we can rely on markets to send us signals that scarcity is imminent. Moreover, those same markets may deal with the scarcity problem by encouraging new discoveries; resource efficiency (now widely called resource productivity); recycling of minerals (but not energy); and substitution by other, more plentiful resources.

The Hotelling theorem

We first need to establish why real prices of minerals and energy should rise through time. This expectation is generated by the theory of exhaustible resources, formulated in its best known way by Harold Hotelling[1] (1931). The way the problem was set up was to ask the question: given a finite stock of some exhaustible resource, at what rate should that resource be depleted? The general answer is that it should be depleted in such a way that the wellbeing (welfare, utility) generated by using the resource is maximised. More analytically, it should be depleted so as to maximise the present value of the flow of consumption goods coming from the use of the resource.

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[1] Harold Hotelling (1895-1973) was an American mathematical economist who made seminal contributions to the theory of the firm (one of the first game theoretic analyses), environmental economics, natural resource economics, demand theory, mathematical statistics (his main subject) etc.

Setting the model up formally[2], we have:

The objective function (the thing we want to maximise):

$$W = NSB_0 + \frac{NSB_1}{1+r} \quad \ldots.[1]$$

The notation follows that in Perman et al (p188-189). W is wellbeing. NSB is ‘net social benefit’, or ‘utility’, 0 and 1 are two time periods, and ? is the discount rate applied to utility over time.

And we have

The constraint:

$$S = R_0 + R_1 \quad \ldots.\ [2]$$

where S is the total stock of the resource, and R is the rate of use of the resource in each period[3]. Notice that the constraint tells us to use up the resource over the two periods after which ‘time ends’. Obviously, we could increase wellbeing if we use up all the resource compared to a situation in which we leave some of the resource behind when time ends.

The demand equation

To complete the model we introduce a demand equation since this permit us to show how the resource price behave over time. This equation is given by

$$P_t = a - bR_t \quad \ldots.[3]$$

i.e. the resource price in any period t is negatively related to the quantity of the resource being used: the higher the price the less is used, the lower the price the more is used.

Perman et al. (p187) show that NSB in any period t is equal to

$$NSB(R_t) = \frac{b}{2}R_t^2 - cR_t \quad \ldots.[4]$$

where the ‘b’ comes from the demand equation, and ‘c’ is marginal cost of extracting the resource. We shall assume c, the marginal cost of extraction, is the same in each period. Essentially, the net social benefit of extracting the resource is given by the

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[2] Try to follow the maths, as it is important to derive results analytically. However, if it defeats you, use the diagrammatic approach.

[3] We follow Perman et al. in using R for resource extraction, but other texts use Q, which is better since R is often reserved for royalty!

area under the demand curve for the resource (the ‘gross consumer surplus’ or gross willingness to pay) minus the cost of extraction[4].

Deriving the Hotelling rule

We now have the ingredients of the Hotelling model. Maximising [1] subject to the constraint [2] involves forming the ‘Lagrangeian’ L as follows[5]:

$$L = NSB_0 + \frac{NSB_1}{(1+r)} - l[S - R_0 - R_1] \quad \ldots.[5]$$

where ? is the ‘Lagrangean multiplier’.

L can now be differentiated with respect to the levels of R in periods 0 and 1, since we want to know what these values are in order to optimise the rate of depletion. First, we substitute [4] in [5] to obtain:

$$L = \left(aR_0 - \frac{b}{2}R_0^2 - cR_0 \right) + \left(\frac{aR_1 - \frac{b}{2}R_1^2 - cR_1}{1+r} \right) - l(S - R_0 - R_1) \qquad \ldots.[6]$$

This looks a bit alarming but has the virtue of now incorporating the demand equation information into the constrained maximisation problem.

Then, differentiating to get the first order conditions:

$$\frac{\partial L}{\partial R_0} = a - bR_0 - c + l = 0 \qquad \ldots.[7]$$

$$\frac{\partial L}{\partial R_1} = \frac{a - bR_1 - c}{(1+r)} + l = 0 \qquad \ldots.[8]$$

It follows (since [7] = [8] = 0) that

$$a - bR_0 - c = \frac{a - bR_1 - c}{(1+r)} \qquad \ldots.[9]$$

From the demand equation we know that

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[4] Their approach makes use of the fact that the area under the demand curve (which can be thought of as a marginal willingness to pay curve) is the gross willingness to pay. Net willingness to pay is consumer surplus, i.e. gross willingness to pay minus expenditure.

[5] If you have forgotten your Lagrangeian calculus, simply note that the first expression on the right hand side is the maximand (the objective function) and the second expression is the stock of resource minus its use over the two periods. Hence the expression inside the brackets is zero. This may seem odd since if it is zero why is it included? Pursue the differentiation of the equation and you will see why.

$P_t = a - bR_t$ and so, substituting this in [9] gives

$$P_0 - c = \frac{P_1 - c}{(1 + r)} \qquad \text{....[10]}$$

THIS IS THE BASIC HOTELLING EQUATION FOR OPTIMAL RESOURCE USE. What does it tell us?

First it tells us that the NET price in period 0 is equal to the discounted value of the NET price in period 1. This result holds over all periods, so that, in general, the net price in period 0 equals the present value of the net price in any other period. What is this **net price**? It is more usually known as the **resource rent**, **resource royalty**, **(marginal) user cost, or depletion premium**.

Second, we can rearrange [9] to work out the **optimal price** in each period. To make things easy, let us suppose c=0, i.e. there is ‘costless’ extraction. Then [9] would be written:

$$P_0 = \frac{P_1}{(1 + r)}$$

which in turn tells us that

$$P_1 = P_0.(1 + r)$$

We can generalise this as follows:

$$P_t = P_0 (1 + r)^t \qquad \text{....[11]}$$

You are likely to see this result in continuous time, rather than discrete time used here. In continuous time it is:

$$P_t = P_0.e^{rt} \qquad \text{....[12]}$$

In a world of costless extraction, the price in any period is equal to the price in the initial period compounded forward at the rate of discount. Put another way, the price of the natural resource rises at a rate equal to the rate of discount (both are in percentage terms).

Third, we can extend the second result to a world where c >0, i.e. where marginal costs are positive. **It is easy to show (from [10] and extended to continuous time) that in a positive cost world the royalty grows at the rate of discount for there to be optimal depletion**:

$$(P_t - c) = (P_0 - c).e^{rt} \qquad \text{....[13]}$$

Fourth, equation [9] is easily rearranged to give:

$$1+r=\frac{P_1-c}{P_0-c}$$

or

$$r=\frac{(P_1-c)-(P_0-c)}{P_0-c} \qquad \text{….[14]}$$

Now the top line on the right hand side is the CHANGE in the royalty between the two periods, and this is expressed as a fraction of the royalty in the first period, i.e. [14] says that **the rate of change of the royalty must equal the social discount rate for there to be an optimal depletion of the natural resource. This is a reformulation of the Hotelling rule in terms of the discount rate.**

So, the Hotelling rule for the optimal depletion of resources can be formulated in various ways: in terms of price, royalty and the discount rate.

Finding the optimal rate of extraction

We now know that the optimal rate at which an exhaustible natural resource should be depleted is given by the quantities (the R's) that are consistent with the optimal price or royalty path derived above. But how can we find the value of R_0 etc.? To find these values we need to use equation [10] (again working in a costless world to make the maths easy), but we also need to recall the demand equation [3] and the overall resource constraint [2]. (Note that Perman et al. derive the values of R_t using a different demand equation to the one they initially use and they operate in continuous time – see Chapter 8, pp 190-193. Here we continue to work with discrete time, just two periods, and the original linear demand curve).

We know that

$$P_1=P_0(1+r)$$
$$P_0=a-bR_0$$
$$P_1=a-bR_1$$
$$S=R_0+R_1$$

Substituting the demand equations into the Hotelling price path equation gives:

$$a-bR_0=\frac{a-bR_1}{1+r}$$

Writing $R_1 = S - R_0$ and substituting it into this equation gives, with some rearrangement:

$$R_0 = \frac{bS + ar}{b(2 + r)}$$

and the same procedure can be used to obtain R_1.

A simulation

Using some hypothetical numbers may help to cement the understanding of the way the model works. Let:
S=2500
b = 0.25, a = 500 to give the demand curve P=500-0.25R
?=0.05 (i.e. 5%)

We have R_0 = [(0.25)(2500)+(500)(0.05)]/[0.05 + (0.25)(0.05)] = 1268. The value of R_1 is immediately derivable from the constraint S = R_0+R_1 to give R_1= 1232.

Notice that the quantity extracted in the second period is less than in the first period.

To find the optimal prices, substitute R in the demand equations:

P_0 = 500-0.25(1268) = 500-317 = 183
P_1 = 500-0.25(1231) = 192

As the model predicts, prices rise through time.

Note that the discounted value of 192 = 192/(1.05) = 183, which is the price in the first period. Again, this is exactly what we predicted: the present value of the price is the same in every period (if costs were positive, the result would be that the present value of the royalty would stay the same over time).

The intuition behind the Hotelling rule

We have established that the optimal rate of depletion of an exhaustible resource is given by the quantities R_0,R_1 (etc for longer periods) which solve for a set of equations in which (a) for costless extraction, the price of the resource rises through time at the rate of discount, (b) for costly extraction, the royalty of the resource rises through time at the rate of discount[6]. What is the intuition behind this result?

The essential characteristic of resources in the ground is that they are capital assets. The owner has two options with respect to unit of the resource: extract it now, or leave it in the ground. If he extracts it now he can sell it and invest the proceeds at ?% p.a. (the rate of discount = the rate of interest in this model). If he leaves it in the ground it looks as if he will gain nothing. But in fact the value of the unit left in the ground rises by the rate of increase in prices (or royalties in the positive cost case), so

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[6] The price of the resource in the ground, i.e. before any extraction is undertaken, is P. The price 'at the wellhead' is P-c.

the owner will be indifferent between extraction and conservation of the marginal unit of the resource if and only if the rate of increase in prices equals the interest rate. But this is the Hotelling optimality condition given in [14] above! Shorn of the complexities, the intuition behind the Hotelling theorem is simple. **It pays to leave resources in the ground so long as the rate of increase in their capital value (their capital appreciation) exceeds the rate of interest. It pays to extract the resource now if the rate of price appreciation is less than the interest rate.**

A practical implication of the Hotelling rule

The Hotelling rule helps us to understand why so many natural resources are threatened. First, although we will revise the rule for renewable resources, we know that many renewable resources are treated as if they are non-renewable. Second, the discount rates applicable to resource extraction contexts are often very high. This may reflect poverty (poor people tend to 'discount the future' more, or face very high interest rates if they want to borrow, e.g. from informal credit markets) but it may also reflect risk as well. With high discount rates the chances are that the rate of price appreciation will be less than the rate of discount rate, inducing rapid extraction and exhaustion of the resource.

The effect of changing parameters on the rate of resource extraction

There are various ways of investigating what happens when there are changes in the factors influencing the rate of resource extraction. The factors that are usually investigated are:

- A change in the discount (interest) rate
- A change in the price of a ('backstop' technology that can substitute for the resource, but which is currently more expensive
- A change in the reserves
- A change in the cost of extraction
- A change in demand
- A change in competitive conditions
- A tax on resource extraction to reflect the social cost of extraction.

To illustrate the effects of these changes, you may prefer to work with the algebra, but Perman et al. follow Pearce and Turner in using a four-quadrant diagram. It is best to familiarise yourself with this diagram and work through the various parameter changes. Here we focus on just one of the quadrants, the one showing the Hotelling price path (the top right hand quadrant in Perman et al. (p194).

A change in the discount rate

Suppose the discount rate increases**.** The effect will be to make the time path of P rise more steeply in Fig 1. However, the original time path, starting at P_0, cannot simply be swivelled about point P_0 and made steeper. If we did this, the price path would be such that the price at which demand falls to zero (P_C, or the 'choke price') or the price at which a new resource takes over (the 'backstop technology) would be reached before the resource is exhausted. This would imply wasting the resource and switching into a new one before the old one was used up. So, not only does the slope of the P(t) line change, so does P_0. It falls to P_1. The intuition is as before. The higher interest rate makes it more attractive to extract the resource, sell it and invest the money. That means the quantity extracted in the early periods increases, so the initial prices are lower. Notice that the resource is completely exhausted earlier than would have been the case. This underlines the previous discussion: high discount rates are not conducive to conservation.

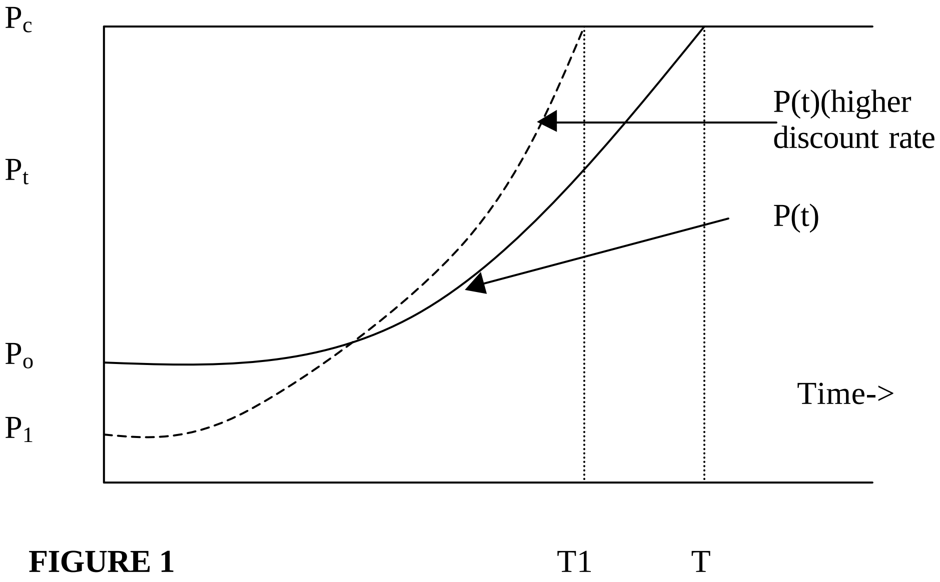

FIGURE 1

A change in the price of a backstop technology

The model works either with a choke price (the price at which demand goes to zero) or with a backstop technology. In the latter case, the price of the backstop technology is initially higher than the price of the natural resource (e.g. nuclear power is more expensive than coal fired energy) but as the coal price rises along the Hotelling path, so the backstop technology switches in when the prices coincide. The argument here is that coal mine owners will know about backstop technology costs, and will therefore plan to deplete the coal resource over a time horizon that is consistent with switching nuclear power in. (In practice, things do not work out like this. Why?).

Suppose the backstop technology gets cheaper due to technological change. Then the situation is as shown in Fig 2. As the backstop price falls the original price path cannot be sustained. This is because it would ‘hit’ the new backstop price, P_c’, before the resource is exhausted at T. As we would expect, if the substitute resource gets cheaper, the current resource will get substituted earlier, so T’ < T. Note that the rate of increase in P is still the same, i.e. equal to ?.

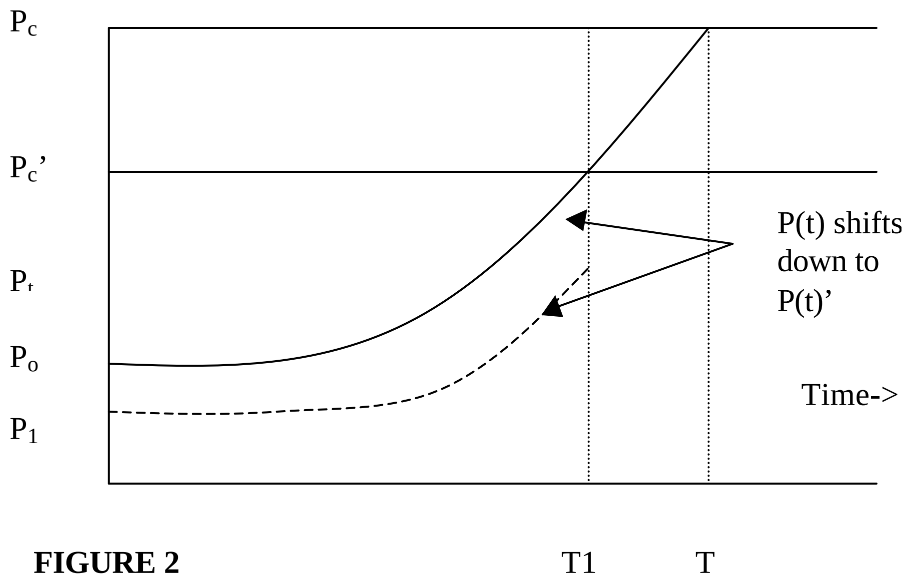

FIGURE 2

A change in reserves

If the quantity of the resource increases and the price path was left unaltered, the price would reach the choke price/backstop price before the newly discovered amount could be used. Thus the resource would not be exhausted: the time path of price must therefore be inefficient. The new optimal price path is given by P(t)’ in Figure 3. It lies below the old price path and gives rise to exhaustion at T’.

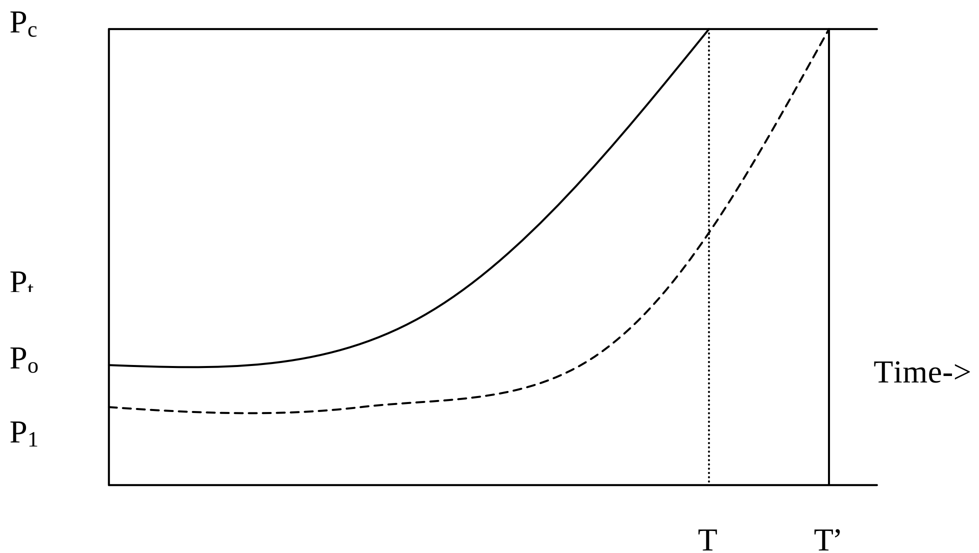

FIGURE 3

In practice, as we saw in the previous lecture, discoveries are made all the time. The effect is to produce repeated downward shifts in the optimal price path, producing a ‘ratchet’ effect (Pearce and Turner, p282; Perman, p199).

A change in extraction cost

Recall that, if extraction costs are zero, it is the price of the resource, P, that rises at the rate ?. If costs are positive (which is obviously the relevant case) it is the royalty (P_0-c) that rises at the rate ?. So, if c falls, the royalty would be bigger at P_0-c’ but would still grow at the rate ?. This would produce a price path that reaches the backstop price before the resource is exhausted. So, P_0 must fall and P_0’-c’ rises at the rate ?, as shown in Figure 4. The resource is exhausted earlier. (Be careful on this one. Perman analyses the case for an increase in cost. Our case is a reduction in cost.)

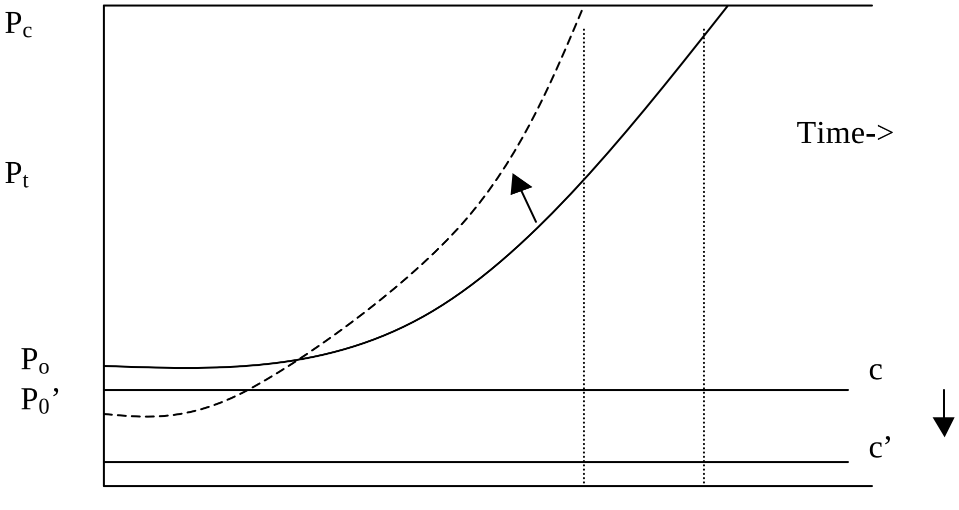

FIGURE 4

A change in demand

A demand increase must shift the price curve P(t) upwards and to the left. The old price path cannot be sustained because, at those prices, the quantities extracted would be higher in each period, and the resource would be exhausted too early (i.e. before the backstop price is reached) compared to the optimal path. Thus, P_0 increases and then increases at the same rate as the old price path, i.e. ?.

The impact of monopoly

All the analysis so far has assumed that perfectly competitive conditions prevail, and we know this is not realistic. You can follow the analysis in Perman (196-7) to see the effects of monopoly but it is also easy to work out by reasoning from first principles. We know that monopolies restrict output relative to a competitive market, so we would expect price to higher and quantities lower. In the natural resource context this means that P_0 will increase. But since the resource stock is fixed, the higher price will reduce extraction in the early periods and increase it in later periods. The resource will be exhausted at a later date. The time path of P is as shown in Figure 5. Notice that the resource is conserved relative to the competitive situation, demonstrating the saying that 'monopoly is the friend of conservation'.

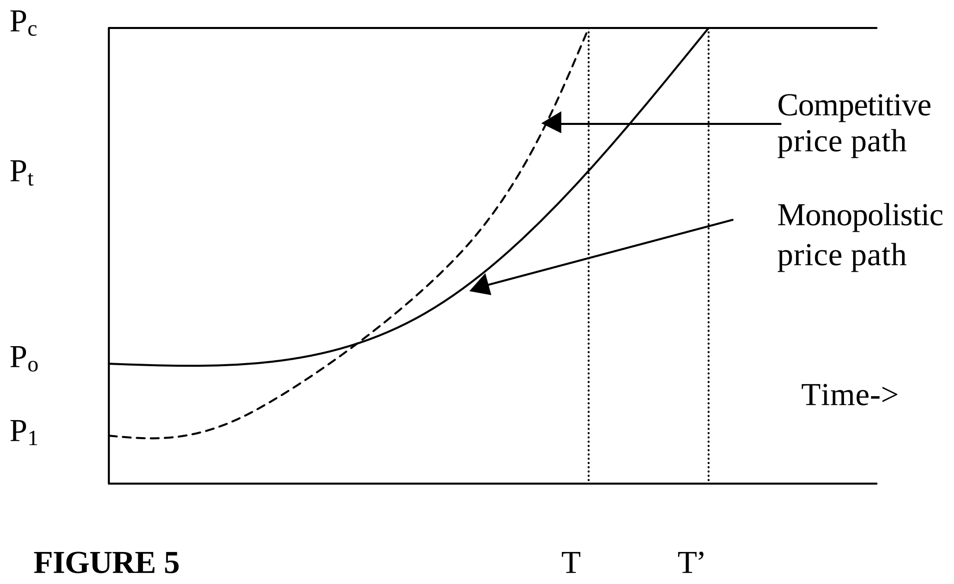

FIGURE 5

Resource taxation

One of the more important implications of the Hotelling model is that **a tax on royalties will have no effect on the path of resource extraction.** The reasoning is simple.

Let

$r_t = p_t - c$ where r is royalty (note that Perman uses a small 'p' which can get confusing)

Then a tax on the royalty at rate k would mean

$r'_t = (1-k)p_t - c$

where r'_t is the after tax royalty. For efficient exploitation, r'_t must rise at the interest rate ?, i.e.

$(1-k)p_t - c = (1-k)p_0.(1+?) - c$

or

$p_t = p_0.(1+ ?)$

But this is the original Hotelling price path. Nothing has happened to change it. The intuition is that, so long as the resource extraction process is underway, rents or royalties can be taxed away theoretically to the point where the mine owner earns only ‘normal’ profits. Notice that this is quite different to arguing that royalty taxes have no effect on exploration: it is easy to imagine someone not starting out on a mine or drilling oil wells if they think the after tax return is too low.