Maximizing social benefit (consumer surplus minus cost)
With constant marginal cost (c), at any period (0 or 1), we want to maximize Net Social Benefit,
NSB(Q) =

$$\int (a - b\cdot Q - c)\,dQ \rightarrow a\cdot Q - \frac{1}{2}\cdot b\cdot Q^2 - c\cdot Q$$

Our two period dynamic optimization become,
Max

$$NSB(Q_0) + \frac{NSB(Q_1)}{1 + r}$$

subject to

$$S = Q_0 + Q_1$$

$$Lg = \left[a\cdot Q_0 - \frac{1}{2}\cdot b\cdot (Q_0)^2 - c\cdot Q_0\right] + \left[\frac{a\cdot Q_1 - \frac{1}{2}\cdot b\cdot (Q_1)^2 - c\cdot Q_1}{1 + r}\right] - \lambda\cdot(S - Q_0 - Q_1)$$

First Order Condition

$$a - b\cdot Q_0 - c + \lambda = 0$$

$$\frac{a - b\cdot Q_1 - c}{1 + r} + \lambda = 0$$

so

$$a - b\cdot Q_0 - c = \frac{a - b\cdot Q_1 - c}{1 + r}$$

or

$$P_0 - c = \frac{P_1 - c}{1 + r}$$

or in multiple period,

$$P_t - c_t = (1 + r)^t\cdot(P_0 - c_0)$$

or in continuous form,

$$P(t) - c(t) = (P(0) - c(0))\cdot e^{r\cdot t}$$

or

$$rent(t) = rent(0)\cdot e^{r\cdot t}$$

Hotelling Rule in general form:
Rent (Price minus marginal cost) will increase at rate of discount.
in costless extraction (special case) price will increase at rate of discount, with cost > 0, it is
rent of the resource that will increase at discount rate.
See Pearce's lecture note on exercise with numerical example!